Teaching Times Tables: A Whole School Approach

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How does identifying and making connections contribute to a deeper understanding of times tables?

**Outcomes:**

- Consider limitations of rote learning and need to develop fluency in concepts as well as facts.

- Plan programme of study based around representation and structure so as to improve pupils ability to recall and use their tables proficiently (not simply rote learning but reasoning their answers).

- Improvement in pupil achievement, attitudes, participation or experience - pupils understand and explore different representations and the structure in times-table and can recall tables facts quickly through understanding.
What is Fluency?

- Not just memorisation of facts, but need to develop fluency in concepts and strategies that underpin facts so retain knowledge and able to apply.

- Reasoning is an integral part of fluency, allows children to make sense of concepts and see how they link together.

- Vast gap between research knowledge and what happens in classrooms when teaching Maths.

When students focus on memorising Maths facts they often memorise facts without number sense, which means they are very limited in what they can do and are prone to making errors (Boaler, 2015).

- The more we emphasise rote memorisation, the less willing children become to think about number relationships.

- Those who learned through strategies achieve ‘superior performance’ over those who memorise, they solve problems at the same speed, and show better transfer to new problems. (Delazer et al, 2005, cited in Boaler, 2015).
What is Number Sense?

The ability to break numbers apart and use the relationships between them to work flexibly and creatively with Maths.

...people can feel overwhelmed because they see Maths as a huge number of facts to remember. By breaking numbers apart and making use of their properties to link them to known facts, the number of facts is reduced. Because it builds understanding, it relieves memory load.

Maths facts are held in working memory, but when students are stressed, such as when they are taking math questions under time pressure, the working memory becomes blocked and students cannot access math facts they know (Beilock, 2011; Ramirez, et al, 2013, cited in Boaler, 2015).

Important difference between high and low achievers in Maths is the level of number sense. Low achievers use inefficient strategies such as counting back or skip counting whereas high achievers use what they know about number relationships to use numbers flexibly (ibid).

Available at: <www.youtube.com/watch?v=yXNG6GKFhQM>
• One of the best methods for teaching number sense and math facts at the same time is a teaching strategy called ‘number talks’, developed by Ruth Parker and Kathy Richardson.

• This involves posing an abstract math problem such as 18 x 5 and asking students to solve the problem mentally and explain how they worked it out.

https://www.youtube.com/watch?v=wxE2Kur4AHc

• Looking a range of ways a calculation can be solved helps students develop understanding of a range of strategies and understand the underlying concepts. It allows them to make links and spot patterns, so they learn to exploit the properties of numbers.
Approach number facts from a range of angles to develop a deep understanding of the relationships and concepts that underlie them.

‘It is better to have five ways of answering one question than one way of answering five questions.’ - Singapore Maths.

‘Children learn at a higher level through variation not repetition’ - Zoltan Dienes.
Need to:

• Use range of representations **throughout** to make connections between number relationships and expose underlying structure.
  • Idea is not to use concrete or visuals to gain answer, but to access understanding of more abstract concepts. What does it help them to notice? Can they then predict and generalise?
  • Visual forms a ‘bridge’ between the concrete and abstract.

• **Deepen understanding of structure through exploration of properties of multiplication and use of range of strategies.** Need to see how numbers can be manipulated in range of ways to exploit and make use of these.

Reason about these relationships using questions in the style provided, so children justify and deepen their own understanding.
Obstacles and Solutions

Obstacle:
- Over-reliance on skip counting.
  - Makes it difficult to access tables facts when not in order.
  - Children struggle to apply tables knowledge.

Solution:
- Start early! Multiple routes to access.
- Practical equipment and represent strategies visually to help children make connections and visualise relationships.
Obstacle:
❖ Key Stage Two children struggle to use flexible multiplication strategies because they do not have the required mental strategies in place from Key Stage One.

Solution:
✓ Identify key skills.
✓ Ensure strategies are automatic for most children by the end of KS1.

They are:
  o Doubling/ halving.
  o Partitioning.
  o Number bonds within ten/multiples of ten.
  o Bridging.

✓ A strong understanding of place value is also required in order for tables facts to be applied with larger numbers and decimals.
• Our experience showed that lack of confidence with these underlying skills significantly impaired a child’s ability to develop alternative strategies for accessing times tables facts. Therefore, these key skills must be given a high priority throughout Key Stage One and Two.
Obstacle:

- Children feel overwhelmed by the number of tables facts to learn.

Solution:

- Make links practically and visually and generalise, e.g. x9.
- Ensure contexts for both scaling and skip counting (e.g. doubling in size as well as groups of 2).
- Revisit key strategies and relationships each year, using similar methods but with different numbers, to allow cumulative learning where familiar structures are used to revisit prior learning, whilst at the same time supporting and scaffolding new learning. This supports working memory.
- Make use of the visual memory and known facts.
### Reduce the facts - understand the relationships

<table>
<thead>
<tr>
<th></th>
<th>1x1</th>
<th>2x1</th>
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<td>3x1</td>
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<tr>
<td>10x1</td>
<td>10x2</td>
<td>10x3</td>
<td>10x4</td>
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</table>

### Skip counting isn’t everything.
To recall rapidly and be able to use in a range of context, commutative aspect is key.
Obstacle:

❖ Children in Key Stage Two over-rely on formal written methods, instead of thinking logically about the numbers and concepts involved.

Solution:

✓ Continue to develop mental methods which can be used for both arithmetic and reasoning questions throughout Key Stage Two, so children learn to make decisions and don’t always default to the formal method.

✓ Delay teaching of formal written methods in Year Three until key mental methods have been explored and understood. (N.b. This is also advised in Year 3 objective).
Obstacle:
- Difficulty applying to fractions or division

Solution:
- Use visual structures and language which explore part whole relationships and make the inverse relationship between multiplication and division clear.

- Use arrays to share as well as group, so the link between sharing and grouping becomes obvious when dividing. Some children have difficulty understanding how counting in threes (grouping) results in finding a third (which is division as sharing). Sharing in an array can help clarify this understanding and support understanding of fractions.

During the project, we found that, although a strategy-based approach worked for all children, it was more successful the earlier it was introduced as there were fewer obstacles to overcome.
Multiplication strategies

Need solid understanding of concept of multiplication as repeated addition before start to try to memorise tables facts.

Need to see and understand lots of representations to fully grasp concept.

Order is important - need to link to and build on what is already known.

Doubling and halving is key.
Skip count - explore patterns, use concrete apparatus, 100 squares, number lines, counting sticks to show lots of different representations BUT DON’T JUST SKIP COUNT.

Investigate connections and patterns - give children chance to predict, justify and generalise.

Jumps on a number line, progressing to jumps on an empty number line. (Important - prepares for counting stick visual). Record repeated addition and use terminology ‘lots of’. Introduce x sign as a quick way to write lots of.  \[5 \times 2\]

Draw \[5 \times 2\] on a number line.
Now draw \[2 \times 5\].

What’s the same? What’s different? Why? Can you show me this fact using an array, repeated addition, a number line? etc

Show me 5 lots of 2

<table>
<thead>
<tr>
<th>in an array</th>
<th>on a number line</th>
</tr>
</thead>
<tbody>
<tr>
<td>as a repeated addition</td>
<td>as a multiplication</td>
</tr>
</tbody>
</table>
The Counting Stick

https://vimeo.com/52472300

https://www.youtube.com/watch?v=yXdHGBfoqfw
2x is doubling.
(Teach doubling before 2x table).

Explore why with arrays.

2 lots of 4 is the same as 4 lots of 2.
Use this array to explain how.

Cameron says this array shows double 4 but Taylor says it shows 2+2+2+2. Who is right?

True or false?
2×3 = 3×2.
Draw/make an array to show.

Need to move away from skip counting in some cases and think logically about what it means.
If a child is working out 6×2 by skip counting, what chance has he/she got of understanding and working out 64×2?
Need to teach in specific order, so build on what already known.

10x - underpins early place value work. Rather than just ‘add a zero’, practical work to understand why this happens and encourage explanations from children.

Link to 100 square, tens frames, counting stick and other concrete equipment. Build in as part of place value work.
5x - halve tens times. (Builds on earlier doubling/halving work).

Lots of exploration practically and visually with arrays, numicon etc. Need to be able to understand 5 as half of a ten and be able to visualise and explain why \(5 \times 6 = 10 \times 3\) so can use \(10x\) to find \(5x\).

Once confident, start to explore in different ways on a number line.

If \(2 \times 10 = 20\), what would \(2 \times 5 = ?\)

If \(10 \times 2 = 20\), what would \(5 \times 2 = ?\)
True or false?
8x5=4x10
3x - counting stick and number line. Builds on earlier doubling and halving strategies.

if 3x3=9, what is 6x3?

Some children may also see it as 5x3 add 3.

Again, also explore practically e.g. with numicon, tens frames, dot formations etc.
Once confident with relationships from skip counting, investigate scaling element. Continue to use arrays to show commutative property. Also, use range of other equipment alongside counting stick/ number line for scaling, so children can begin to deepen understanding and generalise.

3x - triple the number by using doubles and bridging/partitioning for last multiple.

Explore practically and visually with arrays, numicon etc but also show on number line.

Which tables fact does this show?
How can you find 3x6?
4x - double 2 times.

Explore practically and visually with arrays, numicon and a range of practical equipment etc but also link this to number line and counting stick. Once they can skip count in 4s and understand the relationship, it is important to investigate where the 4x appears on counting stick in other tables and begin to generalise, so they can make decisions about the best strategy to use.

Which tables fact does this show?

If 2x3=6, what would 4x3=?

3 x 4 = ____ x 2.

(3 x 2) + (3 x 2).
True or false?
$3 \times 4 = 6 \times 2$.

Show me with cubes.

Show me on a number line.

___ fours = 8 twos. Explain why.
9x - find 10x and take one multiple off. Builds on earlier work, subtracting from a multiple of ten.

if 10x3=30, what is 9x3?

If 10 jumps = 40, what would the times table be?
So what would 9 jumps = ?

Again, also explore practically e.g. with numicon, tens frames, dot formations etc.
11x - 10x plus another multiple.
Explore practically then on number lines, counting sticks etc.

if 10x3 = 30, what is 11x3?

12x - 10x plus double the multiple.

if 10x3 = 30, what is 12x3?

If we changed the 10th multiple to 40, then what would 11x be? Etc
Can you show it in a different way?
8x - double 4x - or find 2x and double it and double again. Builds on earlier doubling.

if 4x3=12, what is 8x3?

Give me two other ways to work this out.

E.g. 8x3 is 6 less than 10x3.

True or false?
8x3 is the same as 4 x 6.
Show me how.
What’s the same? What’s different?
6x - double 3x. Builds on earlier doubling.

If $3 \times 3 = 9$, what is $6 \times 3$?

Some children may also see it as $5 \times 3$ add 3.

Again, also explore practically e.g. with numicon, tens frames, dot formations etc.
7x - usually the hardest, but worth remembering that all of the 7x facts except 7x7 can be quickly worked out using the previous strategies and the commutative law, i.e. that 8x7 can be worked out as 7x8. However, to give another strategy: 7x, add together 5x and 2x. Ensure lots of practical work and visualisation to become automatic. E.g. 7x7

7 x 5 = 35
7 x 2 = 14

So 7x7 = 49
Key Stage One
Key skills

Doubling
Halving
2d+1d addition - partitioning and bridging
Subtraction from a multiple of ten
Tripling
Doubles partitioning.

Partitioning

\[8 + 8\]

\[5 + 3 + 5 + 3\]

\[10\]

\[6\]
Doubles - bridging.

\[6 + 6\]

\[\begin{array}{c}
\begin{array}{c}
+4 \\
\hline
6 \\
10 \\
12
\end{array}
\end{array}\]
12 + 5
10 + 5 + 2

So what would 15 + 2 be?
Halving partitioning.

\[ \frac{1}{2} \text{ of } 16 = \]

\[ 10 + 6 \]

\[ 5 + 3 \]
How many?

30 - 2

What would 40 - 2 = ?
Why?

True or false?
60 - 2 = 58.
Explain your thinking.

Can 50 - 2 = 58.
Why not?
Explain your thinking.
What is Multiplicative Thinking?
A family has £96.00 to spend at the adventure park. Each ride costs £4.00.

How many rides can the family go on?

They think …

Frank

I need to find out how many times I can take £4.00 away from £96.00

96
-4
92
-4
88
-4
84
etc. …

Sayma

There are 25 lots of £4.00 in £100, £96.00 is £4.00 less, so…

25 \times 4 = 100
100 – 4 = 96
So
24 \times 4 = 100
...

Which child is thinking multiplicatively?

Discuss each strategy. Are they both thinking multiplicatively?

(Taken from Peto, S.(2018).)
What is the difference between thinking additively and thinking multiplicatively?

One way of trying to articulate:

1. $1 \times 4 = 4$
2. $2 \times 4 = 8$
3. $3 \times 4 = 12$
4. $4 \times 4 = 16$
5. $5 \times 4 = 20$

Looking down the column - + 4 to the previous amount

Looking across a row - the multiplicand has been made 4 times bigger

How often do we think multiplicatively / provide structures to support children thinking in this way?

(Taken from Peto, S. (2018).)
Understanding alternative contexts.

Multiplication is about scaling.

Year 3:

To solve problems ... including positive integer scaling problems and correspondence problems in which \( n \) objects are connected to \( m \) objects

(Taken from Peto, S.(2018).)
How can we teach children to think multiplicatively?
How can these be embedded within teaching and learning?

Need to use properties of multiplication when planning for learning activities.

- What representations best expose the structure of these properties?
- What contexts lend themselves to these concepts?

(Taken from Peto, S. (2018).)
To solve scaling problems

- What does this mean?
- How is it introduced?
- What are the possible difficulties?

Link it back to arrays, number lines etc.

(Taken from Peto, S.(2018).)
Scaling

(Taken from Peto, S.(2018).)
Need to explore properties of multiplication.

The commutative property:

\[ 3 \times 4 = 4 \times 3 \]

Arrays show commutative property of multiplication. Needs to be investigated, represented and explained with practical apparatus.

The distributive property:

(Taken from Peto, S. (2018).)
Some cards are arranged in order. What could the missing card be? What couldn’t it be? Explain why.

$$3 \times 5 \quad \square \quad 8 \times 5$$

$$7 \times 5 > \square$$

Which tables facts could this show?

- $$4 \times 5 + 4 \times 5$$
- $$2 \times 20$$
- $$2 \times 2 \times 10$$
- $$4 \times 10$$
- $$4 \times 2 \times 5$$
- $$4 \times 5 \times 2$$
The associative property:

You could have worked it out as:

- $4 \times (3 \times 5)$  [4 layers of $3 \times 5$]
- $(4 \times 3) \times 5$  [5 layers of $4 \times 3$]
- $(4 \times 5) \times 3$  [3 layers of $4 \times 5$]

Any way in which you group the factors multiplicatively leaves the product invariant (unchanged).

(Taken from Peto, S.(2018)).
Cube A and cuboid B have the same volume.

Calculate the missing length on cuboid B.

\[ 6 \times 6 \times 6 = 6 \times 4 \times ____ \]

\[ 6 \times 6 \times 6 = 6 \times 4 \times ____ \]

\[ 36 \]

\[ 216 \]

\[ 4 \times 6 = 24 \]

\[ 24 \) \]
### 3x Table: Relationship Between Repeated Addition and Skip Counting

<table>
<thead>
<tr>
<th>Notes</th>
<th>Concrete</th>
<th>Visual to Support Abstract</th>
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<tbody>
<tr>
<td>Although the 3x table is covered in Year 3, counting in threes is covered in Year 2 and there have been questions on Year 2 SATs papers testing the 3x table. Therefore, it makes sense to cover it in Year 2 and revise it in Year 3. Ensure examples which include multiplying by 1 and 0 are included.</td>
<td>Represent and solve problems involving repeated addition of threes using concrete apparatus. Write a repeated addition to show this. How else could we say it? $3+3+3+3+3=4+3$</td>
<td>Colour a 100 square to show the pattern of threes. Can you spot any patterns?</td>
</tr>
<tr>
<td>Can you show this with cubes? Place cubes/numicon/Cuisenaire rods along a counting stick and mark in threes.</td>
<td>Use counting stick methods (see page 7) to explore the relationships between multiples of 3 and find more efficient ways to derive them using underlying skills and known facts. What do you think $11\times3=?$ $12\times3$? How can $10\times3$ help you find $5\times3$? $9\times3$? Make sure the relationship between the ninth and tenth multiple is focused upon particularly and discuss whether this strategy would work for other multiples. E.g. Could you use it for $9\times4$? $9\times6$? Model and write repeated addition and multiplication calculations. Begin to derive the division facts using counting stick. E.g. How many threes in 15? Use an empty counting stick to link facts together and revise other tables. E.g. Point to the tenth multiple and ask if this is 30, what would $9\times$ be? What about if the tenth multiple was 50? 100? 20?</td>
<td>Use counting stick relationships to find missing numbers on empty number lines. Draw number lines to solve calculations and problems. What tables fact does the number line below show? Can you use this to find $8\times3$? Rory says it also shows $2\times3+2\times3$. Is he right? Match repeated addition and multiplication calculations to number line visuals and vice versa. Make up a problem that a calculation or visual could show. Gemma drew this number line to show $4\times3$. What is her mistake?</td>
</tr>
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</table>
### 3x Table: Investigating Relationships

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<th>Notes</th>
<th>Concrete</th>
<th>Visual to Support Abstract</th>
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<tr>
<td>Use language related to scaling as well as skip counting, E.g. 4x3 can be 4 lots of 3 or 4 three times or triple four. Tripling is important, as it helps to show the commutative property and encourages children to think about tables fact from different perspectives. Make arrays (see page 22) to show different multiplication/repeated addition calculations. Use them to discuss the commutative property. E.g. Discuss how it can show both 6x3 and 3x6, 6+6+6 or 3+3+3+3+3+3. Match calculations to concrete representations and to visuals. Extension: What else could it show? (2x9, 9x2). Show me threes threes and another three threes. How many threes have you got? How could we write a calculation to show this?</td>
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</table>

| Make a bar model/part whole model to show 4x3. How many parts will you have? How many in each part? What will the whole be? Now make one to show 3x4. What’s the same? What’s different? Can you make up a problem for each representation? Can you use the bar model above to help you work out 8x3? 2x3? What would your bar model look like if you wanted to show 4 three times/three times as many? Investigate relationships practically with discussion and prediction of what will happen when numbers are changed, doubled etc. Combine in part whole models and bar models practically. |

| Show 6 lots of 3. Predict 12 lots of 3, 3 lots of 37? Prove it using unifix/numicon. Show it on a 100 square/counting stick/number line. |

<table>
<thead>
<tr>
<th>Reasoning</th>
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<tbody>
<tr>
<td>• True or false: 6x3&gt;2x3. Explain how you know.</td>
</tr>
<tr>
<td>• 3x10=______. What could go there? What couldn’t go there?</td>
</tr>
<tr>
<td>• What could go in the middle: 3x3 ____ 7x3? Which of these could not go in the middle: 5x3, 3x4, 3x2. Explain how you know.</td>
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</table>

<table>
<thead>
<tr>
<th>Show 3x4</th>
</tr>
</thead>
<tbody>
<tr>
<td>In an array</td>
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<td>In a part whole model</td>
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Use visuals to solve missing number problems and investigate the relationship between multiplication and division. Write multiplication and division sentences represented by visuals. What do you notice?

2x3=6 4x3=12 Which calculation would come next? Explain.

Show a calculation. What could it mean? E.g. 3x8 = 8 lots of 3, 3 lots of 8, 8 three times, 3 eight times, 3 multiples of 8 etc. Discuss different ways to solve. Practise tripling, which makes use of doubling and bridging.

E.g. e.g. 3x8 = 8+8+8 16 +8
Key Messages

Reduce the facts by:
- Making connections.
- Exploring properties.
- Using concrete and visuals.

Embed key skills early and continue to use.

Develop, use and discuss a range of strategies and representations, so properties and relationships are fully understood.
References:

Association of Teachers of Maths, 2009. *Times Tables in Ten Minutes*. [online video]. Available at: <https://www.youtube.com/watch?v=yXdHGBfoqfw>

Boaler, J. (2015). Fluency Without Fear: Research Evidence on the Best way to Learn Maths Facts [online] [pdf.]. Available at: <https://www.youcubed.org.evidence//fluency-without-fear>


Youcubed at Stanford (2014). From Stanford Online’s ‘How To Learn Math for Teachers and Parents’: Number Talks. [online video]. Available at: <https://www.youtube.com/watch?v=yXNG6GKFhQM>